iPARAS: Incremental Construction of Parameter Space for Online Association Mining*

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Abstract

Association rule mining is known to be computationally intensive, yet real-time decision-making applications are increasingly intolerant to delays. The state-of-the-art PARAS solution, a parameter space framework for online association mining, enables efficient rule mining by compactly indexing the final ruleset and providing efficient query-time redundancy resolution. Unfortunately, as many association mining models, PARAS was designed for static data. Modern transaction databases undergo regular data updates that quickly invalidating existing rules or introducing new rules for the PARAS index. While reloading the PARAS index from scratch is impractical, as even upon minor data changes, a complete rule inference and redundancy resolution steps would have to be performed. We now propose to tackle this open problem by designing an incremental parameter space construction approach, called iPARAS, that utilizes the previous mining result to minimally adjust the ruleset and associated redundancy relationships. iPARAS features two innovative techniques. First, iPARAS provides an end-to-end solution, composed of three algorithms, to efficiently update the final ruleset in the parameter space. Second, iPARAS designs a compact data structure to maintain the complex redundancy relationships. Overall, iPARAS achieves several times speed-up on parameter space construction for transaction databases comparing to the state-of-the-art online association rule mining system PARAS.

Keywords: Association Rule Mining, Parameter Space Model, Redundancy Resolution

1. Introduction

With the advent of computers and means for mass digital storage, companies increasingly gather huge data from various sources. Mining useful information and helpful knowledge from large databases has evolved into an important research area in recent years. Mining of associations, correlations and other meaningful patterns within the dataset is critical for applications ranging from market basket analysis and web usage mining to fraud detection and catastrophic weather forecasting. However, association rule mining is known to be com-

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1. PARAS system can be accessed at http://paras.cs.wpi.edu.

putationally expensive. Mining systems with lagged responses risk losing a user’s attention, more importantly, are often unacceptable in mission critical applications. Worse yet, association rule mining algorithms are parametrized, meaning that data analysts often need to perform numerous successive trial and error iterations and compare the results produced by varying parameter configurations until a satisfactory result is found. Poor selection of parameter values may lead to failure in finding genuine association rules.

In an online mining system, an end user should be able to rapidly iterate over possibly a variety of parameter settings and examine the corresponding association rules matching these settings in an interactive session (Lin et al. (2013)). To provide fast response time essentially for a truly interactive experience, the expensive computations must be performed a priori as in a precomputation step. Furthermore, effective data structures and access methods must be designed to support instant retrieval of the rules for any desired setting. Put differently, not only computation overhead for rule computation but also excessive I/O costs for rule look-up must be avoided at run-time.

Figure 1: 2-D Parameter Space

Figure 2: State-of-the-art

In general, association rule mining is a two-step process: in the first step, the frequent itemsets are found, from which association rules are induced in the second step. As shown in Figure 2, POQM (Aggarwal and Yu (2001)) precomputes the primary itemsets with a low minimum support and stores them in a lattice structure, called Adjacency Lattice, in an offline step. At runtime, rules are then generated from the lattice upon the user request for a specified parameter setting. Further, redundant rules can be eliminated by request during this online generation process. Unfortunately, the change of even a single parameter requires user to run rule generation algorithm again from scratch.

PARAS (Lin et al. (2013)) extends this approach by including rule generation into the offline process. Not only are the rules that satisfy a low primary support and primary confidence precomputed, but also the redundancy relationships among the rules are encoded. A parameter space, in the context of rule mining, consists of an n-dimensional space of parameters such as support and confidence. The precomputed rules are compactly indexed in this parameter space by their parameters for subsequent interactive rule exploration. PARAS has been shown to outperform the state-of-the-art online association rule mining systems by several orders of magnitude in terms of the online query processing speed.

However, both frameworks are designed to handle static datasets only, rendering them ineffective for transaction databases. In transaction databases, new transactions are ap-
pended as time advances. This may introduce new rules as well as invalidate some existing rules. Worse yet, this might lead to dramatic changes of corresponding redundancy relationships among the rules over time. We observed, and verified in the retail dataset (Brijs et al. (1999)) that daily transaction updates may at times affect a small portion of the dataset as well as the precomputed parameter space. Simply re-mining from scratch may waste a huge amount of computational resources and cause tremendous delays. Thus, the maintenance of the parameter space for dynamic datasets is a critical problem to be tackled.

Motivating Example: By analyzing daily online transaction logs, a retail analyst at Amazon may identify products that are frequently purchased together. Such products can be placed together on amazon.com, made into bundled offers or used for recommendations when users search for one of the products. The system can incorporate the updates made by the customers’ daily transactions and can seamlessly update the existing patterns as well as identify new patterns in its e-commerce transaction store.

The Proposed iPARAS Approach. An interactive parameter space system, capable of not only answering mining queries at near real time but also accommodating data updates efficiently, ensures that insights and results are always delivered based on the most up-to-date knowledge for fresh decision making. To dynamically update the pre-stored information, POQM could make use of existing incremental frequent itemset mining (FIM) algorithms (Cheung et al. (1996), Ayan et al. (1999), Cheung and Zaïane (2003), Koh and Shieh (2004), Leung et al. (2005)). We propose to extend the PARAS framework with strategies for maintaining the parameter space up-to-date even over dynamic transaction databases. Our proposed model iPARAS utilizes the previously computed results indexed in parameter space to update the rules and the redundancy relationships in the parameter space. iPARAS features two innovative techniques. First, iPARAS provides an end-to-end solution, composed of three algorithms, to efficiently update the rules maintained in the parameter space. Second, iPARAS designs a compact data structure to maintain the complex redundancy relationships. Overall, iPARAS achieves several times speed-up for the parameter space construction for transaction databases compared to the state-of-the-art online association rule mining system PARAS.

2. Preliminaries

2.1. Foundation: The Parameter Space Model

The parameter space model (Lin et al. (2013)) forms the foundation of our proposed iPARAS framework. A parameter space as depicted in Figure 1, in the context of rule mining, consists of an n-dimensional space of parameters such as support and confidence. A parameter setting is represented as a point located within this space. The key observation is that many rules may map to the same location and thus can be compactly indexed by their location. This finite number of points divide the parameter space into several non-overlapping regions, called stable regions. The ruleset valid for any possible location within a stable region remains unchanged, whereas rulesets valid for two locations not in the same stable region are guaranteed to be distinct. The parameter space model supports instant response to user’s mining request and reveals the overall distribution of rules within the space of interesting parameters so that trial-and-error selection of parameters can be avoided. In addition, to filter out redundant rules for presenting succinct mining results to users, the redundant
relationships among the rules in parameter space are precomputed and encoded using a compact representation, called *dominating location*. In the online phase, redundant rules can be quickly identified and filtered out by comparing their dominating locations to user specified parameters.

### 2.2. iPARAS Framework Overview

To efficiently update rules in the parameter space upon changes of the raw data, while preserving instant responsiveness, we design the *iPARAS* framework, as depicted in Figure 3. *iPARAS* consists of two phases: (a) offline parameter space maintenance and (b) online rule exploration. *iPARAS* adapts *PARAS* for the initial parameter space construction from raw data and adopts *PARAS* to support the powerful online interactive rule exploration.

![Figure 3: The iPARAS framework overview](diagram)

The **Rule Generator** operates in two modes, namely, **Generate** and **Update**. The **Generate** mode is designed for processing the full dataset initially. In general, the association rule mining is a two-step process: (a) **Frequent Itemset Mining** and (b) **Rule Induction**. To perform the above tasks for rule generation, we adapt *FP-Growth* (Han et al. (2000)) and *GenerateRules* (Aggarwal and Yu (2001)) respectively. The second mode, **Update**, has been added to process the future batch updates. To efficiently update the parameter space, we design a novel incremental frequent itemset mining algorithm, called *iFIM* and a rule updating technique.

The **Redundancy Resolver** captures and maintains the redundancy relationships among the rules such that, if desired by the user, redundant rules can be efficiently filtered out for any given parameter settings. We optimize the redundancy resolution introduced in *PARAS* by leveraging our proposed *Adjacency Index*. In **Abstract** mode, this improved strategy is used to compute the dominating locations for the rules generated from the initial dataset. The **Update** mode updates the redundancy relationships among the rules in the adjusted parameter space.

The **Parameter Space Constructor** takes the up-to-date ruleset and the corresponding redundancy information to construct the parameter space. After the construction is completed, the user then is able to explore the parameter space with instant responses through the **Rule Explorer** to find interesting rules.
3. Offline Parameter Space Maintenance

3.1. Rule Generation & Updating

We introduce our Rule Generator module of \textit{iPARAS} that provides an efficient solution for rule induction and updating. The first task of this module is to generate and update frequent itemsets. Apriori (Agrawal and Srikant (1994)), FP-tree (Han et al. (2000)), Eclat (Zaki et al. (1997)) and their variations could all be used to generate frequent itemsets in a database. However, it has been proven (Hunyadi (2011), Borgelt) that the FP-tree solution outperforms Apriori because it requires fewer scans over the database and also the execution time is not wasted in re-producing candidate items at each step. As FP-tree can be applied to larger datasets and reduces I/O computation costs significantly, \textit{iPARAS} now adopts this method to generate frequent itemsets for the original dataset. Thereafter, with each update it is maintained to accommodate new transactions efficiently. Frequent itemset mining involves two phases. In the first phase the FP-tree is constructed while the second phase, known as FP-Growth (Han et al. (2000)), derives the frequent itemsets from FP-tree.

Consider the example of database given in Table 1 with 10 transactions. The minimum absolute support threshold is 2. In the first pass, the database is scanned to get the frequency counts of each item. Items are sorted in descending order of frequency. The transactions with now sorted frequent items are shown in Table 1 (c). The header table is built which contains the sorted frequent items and their frequency and link to the first node in the FP-tree. FP-tree and header table for the database of Table 1 is shown in Figure 5.

After the FP-tree has been constructed, the mining process FP-Growth is executed which finds all frequent itemsets directly from the FP-tree. It is a recursive process that
works bottom up according to the header table. A conditional FP-tree is generated for each item and frequent itemsets are derived recursively from conditional FP-trees. Once frequent items are generated, they are represented in an Adjacency Lattice proposed in (Aggarwal and Yu (2001)) for rule generation process (see Section 3.1.5). The Lattice built on the frequent items for the example database is illustrated in Figure 6.

3.1.1. Incremental Frequent Itemset Mining (iFIM)

The FP-tree on the original database must be built as described in the previous section before newly incoming transactions can be processed. During the construction of the FP-tree the inverted index is created which contains the transaction number for each item. When new transactions are added, iFIM processes them to update the tree in such a manner that the resulting tree has a compact structure and assures that the frequent itemsets are generated from tree efficiently. The algorithm for updating the FP-tree is introduced in Algorithm 1, with the Sort Header procedure adopted from (Koh and Shieh (2004)).

Algorithm 1: iFIM Algorithm

```
Input: FPTree, inverted_index, header, suppI, D+
Output: \{FPTree+\} // Updated FPTree
begin
  foreach Transaction T_i in D+ do
    foreach Item a_j in T_i do
      if a_j present in header then
        header[a_j].count++;
      else
        rescan_vector[a_j].count ++;
      delete_Item(FPTree, header, suppI);
      rescan_Item(header, inverted_index, rescan_vector, suppI);
      sort_header(FPTree, header);
    foreach Transaction T_i in D+ do
      process the transaction
```

Algorithm 2: Sort Header

```
Input: FPTree, header
begin
  Apply bubble sort on header;
  if item X, Y in header are to be exchanged then
    foreach node X, Y in FPTree do
      if node X.count > node Y.count then
        insert new node X' as child of X parent;
        X'.count=node X.count - node Y.count;
        X'.children= X.children- node Y;
      else
        node X.count=node y.count;
        exchange parent & child links of node X and Y;
```

Algorithm 3: Rescan Item

```
Input: header, inverted_index, rescan_vector, suppI
begin
  foreach Item a_i in rescan_vector do
    if rescan_vector[a_i].count >= suppI then
      get transaction number from inverted_index ;
      insert item a_i in header;
      process the transaction;
```

Algorithm 4: Delete Item

```
Input: FPTree, header, suppI
begin
  foreach Item a_i in header do
    if header[a_i].count < suppI then
      foreach node X corresponding to item a_i do
        assign node X children to X parent;
        if parent has two child with same item then
          merge nodes and add node counts;
          delete item a_i from header;
          delete node X from tree;
```
3.1.2. Incremental Frequent Itemset Mining Example

Table 2: The three new transactions and frequency count of each item in new and updated database.

<table>
<thead>
<tr>
<th>TID</th>
<th>Items</th>
<th>Item</th>
<th>Supp in $D'$</th>
<th>Supp</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>d,f,g</td>
<td>a</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>d,g,i</td>
<td>b</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>13</td>
<td>f,h,j</td>
<td>c</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>d,f,g,i</td>
<td>d</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>15</td>
<td>a,b,c</td>
<td>e</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>d,g,i,j</td>
<td>f</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Let $D$ be the database shown in Table 1 and let minimum support threshold be 2. For the original database the inverted_index is built (see Figure 4). Sorted transactions according to frequencies is shown in Table 1 and corresponding FP-tree in figure 5. Items f, a, b, d, c and e are frequent in $D$. Let $D^+$ as shown in Table 2 contain 3 transactions. Let Incremental support ($suppI$) be 3. The new transactions are scanned to get the counts of each item. The header table is updated and update status of each changed item is set. The updated count of each item is determined. In Figure 7, item c’s updated support is below threshold so it is deleted.

Items that have become frequent after the update yet were infrequent before are processed next. Item g’s updated support is now above the threshold, but was not present in the original FP-tree. It is inserted in rescan_vector and header table. Transactions containing any of the items in rescan_vector are obtained from inverted_index. Only those selected transactions are indexed rather than scanning the whole database again. This avoids unnecessarily scan of the whole database again. For item g only transaction 4 needs to be rescanned. After deletion of item c and rescanning item g, the resulting tree is depicted in Figure 7.

![Figure 7](image-url)  

**Figure 7:** The header table and FP-tree after deleting item c and rescanning transactions.

Next, bubble sort is applied on the header table. In our running example in Figure 8, Item a is exchanged with Item d. The nodes in FP-tree are exchanged and merged accordingly. The items in each new transactions are ordered according to the updated descending order of frequencies of items. They are then processed in same way as FP-tree is constructed originally to get updated FP-tree as shown in figure 9.

![Figure 8](image-url)  

**Figure 8:** The header table and FP-tree after exchange process.
3.1.3. Modified Growth Algorithm

Figure 9: Final header table and FP-tree after all new transactions are processed.

We design a modified growth function that generates frequent itemsets from this updated FP-tree avoiding to generate all frequent itemsets from scratch. It recursively processes those items in the header table whose counts have been updated and ignores those that are not modified.

**Lemma 1** If the candidate itemset $X$ is not affected by the current update over the database, then no extension $X + I_j$ of $X$, where $|I_j| > |I_k|$ for any $I_k$ in $X$ must be changed.

According to Lemma 1, for any item whose count is not changed by current update, any itemset in which it occurs cannot have changed. Thus it can be ignored. A conditional FP-tree is generated for each modified frequent item. From this tree, frequent itemsets are recursively derived. The conditional FP-tree of each item in turn only contains the modified items thus reducing the size of the conditional tree. Consider the updated FP-tree shown in Figure 9. Conditional FP-tree for item g is shown in Figure 10 where conditional FP-tree does not contain item b. The modified growth function then modifies the Adjacency Lattice to reflect the changes for each updated frequent itemset. Any frequent itemset that is below the threshold is removed, while new itemsets are added into the lattice. The updated lattice is shown in Figure 12.

3.1.4. Adjacency Index

Figure 11: Adjacency Index $L$

Figure 12: Updated Adjacency Index $L'$

In this section, we describe our proposed data structure *Adjacency Index* which extends the capabilities of *Adjacency Lattice* (Aggarwal and Yu (2001)) to facilitate rule updating

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2. Given a set of items $I$, an itemset $X + Y$ of items in $I$ is said to be an extension of the itemset $X$ if $X \cap Y = \emptyset$ (Agrawal et al. (1993))
and redundancy resolution in iPARAS. **Adjacency Lattice** is a compact data structure to store the frequent itemsets. It is also used for association rule induction. An itemset $X$ is said to be adjacent to an itemset $Y$, if $X$ can be obtained from $Y$ by adding just one item. In that case, $X$ and $Y$ are connected by a one-directional link from $Y$ to $X$. Each node in the **Adjacency Lattice** represents a frequent itemset. Our proposed **Adjacency Index** structure extends the capability of the **Adjacency Lattice**. As shown in Figure 11, in each node which represents an itemset $X$, there are two embedded indices, namely $\text{Index}^{\text{ant}}$ and $\text{Index}^{\text{all}}$. $\text{Index}^{\text{ant}}$ indexes the rules whose Antecedents $\subseteq X$, while $\text{Index}^{\text{all}}$ indexes the rules with (Antecedents $\cup$ Consequents) $\subseteq X$. In addition, the one-directional links among the itemsets are replaced by bi-directional links to support both top-down and bottom-up traversal functions.

### 3.1.5. Rule Induction for Initial Dataset

**Table 3:** Association Rule Table

<table>
<thead>
<tr>
<th>Rule</th>
<th>S</th>
<th>C</th>
<th>Rule</th>
<th>S</th>
<th>C</th>
<th>Rule</th>
<th>S</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1 \equiv (a \Rightarrow d)$</td>
<td>4</td>
<td>4/6</td>
<td>$R_{10} \equiv (d \Rightarrow af)$</td>
<td>3</td>
<td>3/6</td>
<td>$R_{19} \equiv (f \Rightarrow c)$</td>
<td>2</td>
<td>2/7</td>
</tr>
<tr>
<td>$R_2 \equiv (a \Rightarrow df)$</td>
<td>3</td>
<td>3/6</td>
<td>$R_{11} \equiv (d \Rightarrow b)$</td>
<td>3</td>
<td>3/6</td>
<td>$R_{20} \equiv (f \Rightarrow d)$</td>
<td>5</td>
<td>5/7</td>
</tr>
<tr>
<td>$R_3 \equiv (a \Rightarrow e)$</td>
<td>2</td>
<td>2/6</td>
<td>$R_{12} \equiv (d \Rightarrow bf)$</td>
<td>2</td>
<td>2/6</td>
<td>$R_{21} \equiv (ad \Rightarrow f)$</td>
<td>3</td>
<td>3/4</td>
</tr>
<tr>
<td>$R_4 \equiv (a \Rightarrow f)$</td>
<td>3</td>
<td>3/6</td>
<td>$R_{13} \equiv (d \Rightarrow f)$</td>
<td>5</td>
<td>5/6</td>
<td>$R_{22} \equiv (af \Rightarrow d)$</td>
<td>3</td>
<td>3/3</td>
</tr>
<tr>
<td>$R_5 \equiv (b \Rightarrow d)$</td>
<td>3</td>
<td>3/6</td>
<td>$R_{14} \equiv (e \Rightarrow a)$</td>
<td>2</td>
<td>2/3</td>
<td>$R_{23} \equiv (bd \Rightarrow f)$</td>
<td>2</td>
<td>2/3</td>
</tr>
<tr>
<td>$R_6 \equiv (b \Rightarrow df)$</td>
<td>2</td>
<td>2/6</td>
<td>$R_{15} \equiv (f \Rightarrow a)$</td>
<td>3</td>
<td>3/7</td>
<td>$R_{24} \equiv (bf \Rightarrow d)$</td>
<td>2</td>
<td>2/4</td>
</tr>
<tr>
<td>$R_7 \equiv (b \Rightarrow f)$</td>
<td>4</td>
<td>4/6</td>
<td>$R_{16} \equiv (f \Rightarrow ad)$</td>
<td>3</td>
<td>3/7</td>
<td>$R_{25} \equiv (df \Rightarrow a)$</td>
<td>3</td>
<td>3/5</td>
</tr>
<tr>
<td>$R_8 \equiv (c \Rightarrow f)$</td>
<td>2</td>
<td>2/2</td>
<td>$R_{17} \equiv (f \Rightarrow b)$</td>
<td>4</td>
<td>4/7</td>
<td>$R_{26} \equiv (df \Rightarrow b)$</td>
<td>2</td>
<td>2/5</td>
</tr>
<tr>
<td>$R_9 \equiv (d \Rightarrow a)$</td>
<td>4</td>
<td>4/6</td>
<td>$R_{18} \equiv (f \Rightarrow bd)$</td>
<td>2</td>
<td>2/7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The second task of **Rule Generator** is to induce and update rules from the generated itemsets. For the **Generate** mode, we adapt the **GenerateRules** (Aggarwal and Yu (2001)) rule induction algorithm and leverage the **Adjacency Index L** to generate the ruleset $\{R\}$ from the initial dataset. The rule generation algorithm forms a rule based on the examination of two itemsets where one is a subset of another. Consider two itemsets $X$ and $Y$, where $X \subseteq Y$, if $Y^{\text{support}}_X$ is larger than the primary confidence threshold, then $R \equiv (X \Rightarrow (Y-X))$ is generated and indexed by $\text{Index}^{\text{ant}}_X$ and $\text{Index}^{\text{all}}_Y$ respectively. To find out all possible pairs to form the rules, for each itemset $Y$, the algorithm **GenerateRule**(Y) performs a Depth-First Search (DFS) to traverse its ancestors $\{X_i\}$. Any item in $\{X_i\}$ is a subset of $Y$. Therefore, $\{X_i\}$ is sufficient for $Y$ to form rules where $X_i$ is the Antecedents. This process is performed on each itemset of **Adjacency Index L**. We modify this algorithm such that whenever a rule is formed, this rule will be immediately indexed by $\text{Index}^{\text{ant}}_{\text{Antecedents}}$ and $\text{Index}^{\text{all}}_{\text{Antecedents} \cup \text{Consequents}}$. Consider the itemset “bd” on $L$. “bd” is its only descendant. When the modified algorithm **GenerateRule**(“bd”) generates the rule $R = (bd \Rightarrow f)$, $R$ will be indexed by $\text{Index}^{\text{ant}}_{\text{bd}}$ and $\text{Index}^{\text{all}}_{\text{bd}}$ in preparation for iPARAS rule updating algorithm and redundancy resolution.
3.1.6. Rule Updating for Batch Update

<table>
<thead>
<tr>
<th>Rule</th>
<th>S</th>
<th>C</th>
<th>Rule</th>
<th>S</th>
<th>C</th>
<th>Rule</th>
<th>S</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_1</td>
<td>4</td>
<td>4/6</td>
<td>R_{10}</td>
<td>3</td>
<td>3/8</td>
<td>R_{20}</td>
<td>5</td>
<td>5/9</td>
</tr>
<tr>
<td>R_2</td>
<td>3</td>
<td>3/6</td>
<td>R_{11}</td>
<td>3</td>
<td>3/8</td>
<td>R_{21}</td>
<td>3</td>
<td>3/4</td>
</tr>
<tr>
<td>R_3</td>
<td>3</td>
<td>3/6</td>
<td>R_{13}</td>
<td>5</td>
<td>5/8</td>
<td>R_{22}</td>
<td>3</td>
<td>3/3</td>
</tr>
<tr>
<td>R_7</td>
<td>4</td>
<td>4/6</td>
<td>R_{15}</td>
<td>3</td>
<td>3/9</td>
<td>R_{27}</td>
<td>3</td>
<td>3/8</td>
</tr>
<tr>
<td>R_6</td>
<td>4</td>
<td>4/8</td>
<td>R_{17}</td>
<td>4</td>
<td>4/9</td>
<td>R_{28}</td>
<td>3</td>
<td>3/3</td>
</tr>
</tbody>
</table>

In Section 3.1.1, we demonstrated how to update the Adjacency Index L into L’ for a given batch update. The operations performed on L are Delete, Insert and Update. Each of these operations will correspondingly trigger certain actions in our proposed rule updating algorithm. In the Adjacency Index L, if an itemset X is:

**Deleted:** All the rules in Index_{X}^{all} should be deleted. Because all rules indexed by Index_{X}^{all} consist of X. For instance, as indicated in Figure 12, “cf” has been deleted from L. It is obvious that the two rules indexed by Index_{c_{X}}^{all}, which are c \Rightarrow f and f \Rightarrow c should be deleted from \{R\}.

**Inserted:** Similar to the rule generation algorithm GenerateRule(Y), AddRule(Y) runs a DFS to traverse Y’s ancestors \{X_i\}. The newly generated rules conform to the template R_i \equiv (X_i \Rightarrow (Y - X_i)) and they will be immediately indexed on L’. For example, for the newly inserted itemset “dg” in L’, two new rules d \Rightarrow g and g \Rightarrow d will be added to \{R\}.

**Updated:** The rules originally formed by X and its descendants \{Y_i\} may be updated or expired due to the change of their support and confidence values. It is also possible to introduce new rules for some R \equiv (X \Rightarrow (Y_i - X)) whose original confidence value was not larger than the primary confidence value. Similar to the GenerateRule(X) algorithm, UpdateRule(X) traverses X’s descendants to examine the above cases and make changes to \{R\}. For instance, the support of “d” being increased will affect the confidence of R_9, R_{10}, R_{11} and R_{13} in Table 3.

3.2. Redundancy Resolution in iPARAS

Aggarwal and Yu (2001) defined redundancy relationships among rules, such that redundant rules, if desired, may be filtered out for presenting succinct result to the user. In particular, redundancy relationships can be of two types, as defined below:

**Definition 1 Simple Redundancy:** Let R \equiv ((A_1 : A_n) \Rightarrow (C_1 : C_m)) be a rule. All rules that simple dominate it conform to the template R^{\gg sim} \equiv (((A_1 : A_n) - (A_v : A_w)) \Rightarrow ((A_v : A_w) \cup (C_1 : C_m))) where (A_v : A_w) \subset (A_1 : A_n). The rule R is simple redundant with respect to the rule R^{\gg sim}.

**Definition 2 Strict Redundancy:** Let R \equiv ((A_1 : A_n) \Rightarrow (C_1 : C_m)) be a rule. All rules that strict dominate it conform to the template R^{\gg str} \equiv (((A_1 : A_n) - (A_v : A_w)) \Rightarrow ((A_v : A_w) \cup (C_1 : C_{m+e}))) where (A_v : A_w) \subset (A_1 : A_n) and (C_1 : C_m) \subset (C_1 : C_{m+e}). The rule R is strict redundant with respect to the rule R^{\gg str}.

Consider R_{22} \equiv (ad \Rightarrow f) and R_2 \equiv (a \Rightarrow df) in Table 3. By Definition 1 above, R_2 is simple dominating R_{22}. The support values of them are guaranteed to be the same.
because $R^{\leq \text{sim}}$ always consists of the same items as $R^{\gg \text{sim}}$. Consider $R_5 \equiv (b \Rightarrow d)$ and $R_6 \equiv (b \Rightarrow df)$ in Table 3. By Definition 2 above, $R_6$ is strict dominating $R_5$. Note that both support and confidence values of $R^{\leq \text{str}}$ are greater or equal to the values of $R^{\gg \text{str}}$.

The rules produced in the parameter space may contain redundancies. However, redundancy relationship is a runtime phenomenon. In other words, the redundancy among rules depends on runtime input parameters, rendering it impossible to eliminate a rule as redundant at an offline step. For instance, let $R_i$ be the only rule that dominates $R_j$ in the parameter space, $R_j$ is said to be redundant if and only if both of them exist in the ruleset that is generated based on a user specified parameter setting. While eliminating redundant rules can only be performed at runtime, the system must isolate as much as possible the redundancy relationships inside the parameter space in the preprocessing phase. Moreover, database updates may trigger changes of the redundancy information, hence the need for an updating mechanism to identify those changes. In the parameter space model, redundancy information can be abstracted and updated compactly as an offline step by utilizing the certain properties (Lin et al. (2013)) of the redundancy relationships. By leveraging the Adjacency Index, our proposed redundancy abstraction method is exponentially faster by a factor of approximately $1.5^n$ comparing to the state-of-the-art PARAS approach, where $n$ is the number of unique items in $D$.

3.2.1. Redundancy Abstraction in PARAS

Lin et al. (2013) identified a compact representation of the rule redundancies in the parameter space model, namely, the simple dominating location and the strict dominating location. Given a parameter setting, to determine whether or not a candidate rule $R$ is redundant, it is sufficient to compare it with only two dominating locations instead of an exponential number of dominating rules. The simple and the strict dominating location are described below.

**Lemma 2 Simple Dominating Location:** For each simple dominated rule $R^{\leq \text{sim}}$, the set of simple dominating rules $\{R^{\gg \text{sim}}\}$ contains a rule $R^{\gg \text{sim}}_i$ closest to the dominated rule $R^{\leq \text{sim}}$, such that $\forall R^{\gg \text{sim}}_k \in \{R^{\gg \text{sim}}\}$, where $k \neq i$, $R^{\gg \text{sim}}_i . \text{conf} \geq R^{\gg \text{sim}}_k . \text{conf}$. The location of rule $R^{\gg \text{sim}}_i$ is called the simple dominating location of $R^{\leq \text{sim}}$, denoted by $\mathcal{L}^{\gg \text{sim}}$.

For a $R \equiv ((A_1 : A_n) \Rightarrow (C_1 : C_m))$, Lin et al. (2013) revealed that $\mathcal{L}^{\gg \text{sim}}$ can be obtained from the candidate rules which conform to the template $R^{\gg \text{sim}} \equiv (((A_1 : A_n) - A_i) \Rightarrow (A_i \cup (C_1 : C_m)))$. Since $R$ and its simple dominating rule $R^{\gg \text{sim}}_i$ share the same support value, the goal here is to find out which candidate has the biggest confidence value. We know that the smaller the antecedents.support, the larger the $R$.confidence. So the problem has been transferred into the problem to find the smallest support value from $\{(A_1 : A_n) - A_i).support\}$. This takes $O(n)$. In the Adjacency Index, $\{(A_1 : A_n) - A_i\}$ are the parents of $(A_1 : A_n)$.

**Lemma 3 Strict Dominating Location:** For each strict dominated rule $R^{\leq \text{str}}$, the set of strict dominating rules $\{R^{\gg \text{str}}\}$ contains a rule $R^{\gg \text{str}}_i$ closest to the dominated rule $R^{\leq \text{str}}$, such that $\forall R^{\gg \text{str}}_k \in \{R^{\gg \text{str}}\}$, where $k \neq i$, $R^{\gg \text{str}}_i . \text{supp} \geq R^{\gg \text{str}}_k . \text{supp}$ and $R^{\gg \text{str}}_i . \text{conf} \geq$
The location of rule $R^{\succ sim}_{i}$ is called the strict dominating location of $R^{\ll sim}_{i}$, denoted by $l^{\succ sim}_{i}$.

For a $R \equiv ((A_1 : A_n) \Rightarrow (C_1 : C_m))$, Lin et al. (2013) showed that $l^{\succ sim}_{i}$ can be obtained from the candidate rules which conform to the template $R^{\succ sim}_{i} \equiv ((A_1 : A_n) \Rightarrow ((C_1 : C_m) \cup C_i))$. Since $R$ and its strict dominating ruleset $\{R^{\succ sim}_{i}\}$ share the same antecedents, the goal here is to find out which candidate has the biggest support value. So the problem has been mapped to the problem of finding the biggest support value from $\{(A_1 : A_n) \cup (C_1 : C_m) \cup C_i).support\}$. This takes $O(e)$ where $e$ is the number of children of $((A_1 : A_n) \cup (C_1 : C_m))$ in the Adjacency Index.

3.2.2. Optimized Redundancy Abstraction in iPARAS for Initial Dataset

For each rule in the parameter space, PARAS takes linear time to obtain its dominating locations. However, how many rules are in the parameter space? Let $n$ be the number of unique items in $D$, then the maximum number of frequent itemsets is $2^n$ and the maximum number of rules is $3^n - 2^n + 1$. Even for some small datasets such as Chess and Mushroom (Newman and Asuncion (2007)), where the number of unique items $n$ are 75 and 119, an immense amount of rules exist in the parameter space, resulting in the high computational costs for abstracting redundancy information. Fortunately, we made the important observation that some rules share exactly the same computation in the process of redundancy abstraction. This provides an optimization opportunity.

Lemma 4 If $R_{i}.antecedents \equiv R_{j}.antecedents$, $\{R^{\succ sim}_{i}\} \neq \emptyset$ and $\{R^{\succ sim}_{j}\} \neq \emptyset$, then there must exist a $R^{\succ sim}_{i}$ located at $l^{\succ sim}_{i}$ and a $R^{\succ sim}_{j}$ located at $l^{\succ sim}_{j}$ sharing the same antecedents.

Based on Lemma 4, we can conclude that if there are two rules having the same antecedents, then they share the same computation for obtaining their simple dominating location. Consider $R_{25} \equiv (df \Rightarrow a)$ and $R_{26} \equiv (df \Rightarrow b)$ in Table 3. $R^{\succ sim}_{25}$ at $l^{\succ sim}_{25}$ is $R_{10} \equiv (d \Rightarrow af)$. $R^{\succ sim}_{26}$ at $l^{\succ sim}_{26}$ is $R_{12} \equiv (d \Rightarrow bf)$. Observe that $R_{10}$ and $R_{12}$ share the same antecedents.

Lemma 5 If $R_{i}.items \equiv R_{j}.items$, $\{R^{\succ str}_{i}\} \neq \emptyset$ and $\{R^{\succ str}_{j}\} \neq \emptyset$, then there must exist a $R^{\succ str}_{i}$ located at $l^{\succ str}_{i}$ and a $R^{\succ str}_{j}$ located at $l^{\succ str}_{j}$ sharing the same items.

Based on Lemma 5, we can conclude that if there are two rules consisting of the same set of items, then they share the same computation for obtaining strict dominating location. Consider $R_{13} \equiv (d \Rightarrow f)$ and $R_{20} \equiv (f \Rightarrow d)$ in Table 3. $R^{\succ str}_{13}$ at $l^{\succ str}_{13}$ is $R_{10} \equiv (d \Rightarrow af)$. $R^{\succ str}_{20}$ at $l^{\succ str}_{20}$ is $R_{16} \equiv (f \Rightarrow af)$. Observe that $R_{10}$ and $R_{16}$ consist of a same set of items. In the Adjacency Index $L$, a node, representing an itemset $X$ where the length of $X$ is $n$, indexes rules which have the same antecedents as $X$ and rules which consist of $X$. Instead of obtaining the simple dominating location for each rule in this first set at a cost of $O(n)$, the computation can be performed only once by leveraging this index. Similarly, $O(e)$ time computation is sufficient to obtain their strict dominating locations for the second set of rules. Considering the fact that the maximum number of rules is exponentially larger than
the maximum number of itemsets. Our optimized redundancy resolution is exponentially faster than PARAS by a factor of approximately $1.5^n$ where $n$ is the number of unique items in the database.

3.2.3. REDUNDANCY RELATIONSHIPS UPDATING FOR BATCH UPDATE

**Algorithm 5: Redundancy Update**

<table>
<thead>
<tr>
<th>Input: $L', {\text{Itemset}}^{\text{changed}}, {R^+}$</th>
<th>Output: ${R^+}$ // Rules with redundancy info</th>
</tr>
</thead>
<tbody>
<tr>
<td>begin</td>
<td></td>
</tr>
<tr>
<td>switch Itemset, do</td>
<td></td>
</tr>
<tr>
<td>case Updated</td>
<td></td>
</tr>
<tr>
<td>forall the Itemset in ${\text{Parents of Itemset}}$ do</td>
<td></td>
</tr>
<tr>
<td>RedundancyAbstraction($\text{Index}^{\text{all}}_{\text{Itemset}}$);</td>
<td></td>
</tr>
<tr>
<td>forall the Itemset in ${\text{Children of Itemset}}$ do</td>
<td></td>
</tr>
<tr>
<td>RedundancyAbstraction($\text{Index}^{\text{antecedent}}_{\text{Itemset}}$);</td>
<td></td>
</tr>
<tr>
<td>case Inserted</td>
<td></td>
</tr>
<tr>
<td>RedundancyAbstraction($\text{Index}^{\text{all}}_{\text{Itemset}}$);</td>
<td></td>
</tr>
<tr>
<td>RedundancyAbstraction($\text{Index}^{\text{antecedent}}_{\text{Itemset}}$);</td>
<td></td>
</tr>
<tr>
<td>Update Process in the “Updated” case;</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 13: An example of Adjacency Index update](image)

Given a batch update to dataset $D$, new rules may be introduced and old ones expired or updated in terms of their *support* and *confidence* values, resulting in corresponding changes of the redundancy information in the parameter space. As we discussed in Section 3.2.2, the redundancy can be abstracted in the *Adjacency Index*. Therefore, changes of the itemsets can be used to identify the possible changes of redundancy information. The update strategies for redundancy information fall into two different cases:

I. The *support* of an itemset in the *Adjacency Index* is updated.

In Figure 13, the support of “a”, “c” and “ac” has all changed. All corresponding changes of the ruleset have been made by the rule updating mechanism as well. Consider the node “ac” on the left side in Figure 13, it is the least frequent parent of “abc”. However, on the right side, “ab” becomes the least frequent parent of “abc” due to increasing frequency of “ac”. Therefore, once an itemset is updated, all its child nodes have to be re-examined for possibly having to update the simple dominating locations for the rules in $\text{Index}^{\text{ant}}$ in each child node. On the left side, “ac” is not the most frequent child of “a”. However, on the right side, “ab” is no longer the most frequent children of “a” due to the increasing frequency of “ac”. Therefore, once an itemset is updated, all its parent nodes have to be re-examined for possibly having to update the strict dominating locations for the rules in $\text{Index}^{\text{all}}$ in each parent node.

II. A new itemset is inserted into the *Adjacency Index*.

Our optimized redundancy abstraction is first executed on this node. Then the above redundancy updating algorithm is performed.
4. Experimental Results

Setup. Experiments are conducted on a OS X machine with 2.4 GHz Intel Core i5 processor and 8 GB of RAM. The system and its competitors are implemented in C++ using Qt Creator with Clang 64-bit compiler.

Datasets. We evaluated the performance of the iPARAS system using synthetic and real dataset benchmarks. The retail dataset (Brijs et al. (1999)) contains 88,163 transactions collected from a Belgian retail supermarket store in 5 months. To better measure the performance of the systems, we replicated the retail dataset 10 times denoted as 10retail. We also generated 6 datasets by the IBM Quest data generator modeling transactions in a retail store. On average, each transaction has 10 items.

<table>
<thead>
<tr>
<th>10retail</th>
<th>T20000k</th>
<th>T5000k</th>
<th>T16000k</th>
<th>T8000k</th>
<th>T4000k</th>
<th>T200k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transactions</td>
<td>881,630</td>
<td>19,663,637</td>
<td>4,916,218</td>
<td>1,573,347</td>
<td>786,700</td>
<td>393,210</td>
</tr>
<tr>
<td>Unique Items</td>
<td>16,470</td>
<td>23,870</td>
<td>7,062</td>
<td>7,599</td>
<td>7,596</td>
<td>7,589</td>
</tr>
<tr>
<td>Size</td>
<td>40.8 MB</td>
<td>1.2 GB</td>
<td>248.8 MB</td>
<td>79.6 MB</td>
<td>39.8 MB</td>
<td>19.9 MB</td>
</tr>
</tbody>
</table>

4.1. Evaluation of Online Processing Time

To evaluate the online processing time, we compared the iPARAS system against the original Apriori, Eclat, FP-growth implementations from (Borgelt) and POQM. The primary support setting for POQM and iPARAS is 0.03% and the primary confidence setting for iPARAS is 15%. Figure 14 and 15 illustrate the mining request processing time for T5000K. First, we fixed the support to a constant value and measured mining request processing times by varying confidence values. In Figure 14, for all of the five alternate algorithms, the mining request processing times decreased with increasing confidence value. As confidence increases, more rules get filtered, producing fewer rules in output. Overall, iPARAS outperformed four competitors by several orders of magnitude. Second, we fixed the confidence value and measured mining request processing times by varying support values. Figure 15 shows that the trend of the five alternatives is similar. iPARAS performed several orders of magnitude better than the four competitors. Overall, iPARAS consistently outperformed Apriori, POQM, Eclat and FP-growth by 4, 4, 3 and 4 orders, respectively.
4.2. Evaluation of Offline Construction Time

To evaluate the offline construction time, we compared the the \textit{iPARAS} system against the state-of-the-art rule mining system \textit{PARAS}. Both \textit{iPARAS} and \textit{PARAS} precompute the frequent itemsets and association rules to construct the parameter space. When new data arrives, \textit{iPARAS} can utilize the previous knowledge for construction while \textit{PARAS} has to start from scratch. First, we used each system to process a large initial dataset. Then we measured the precomputing time of each system given a relatively small batch update.

![Figure 16: Construction time for one update](image1)

![Figure 17: Ratio of construction time](image2)

For the \textit{10retail}, we divided the dataset into two sets: one with 90 percent of the dataset, for initial construction, and one with 10 percent of the dataset, for randomly batch update generation. We measured the precomputing times by varying the size of the updates. For both systems, the primary support is 0.1\% and the primary confidence is 10\%. In Figure 16, \textit{iPARAS} achieved about 16-fold, 10-fold, 7-fold and 5-fold speed-up of overall offline precomputing time compared to \textit{PARAS} with respect to the update size of 1\%, 2\%, 4\% and 8\%. The differences of time consumed decrease with increasing size of the batch updates. Figure 17 shows the ratios of the running time of modules in \textit{iPARAS} to the running time of corresponding modules in \textit{PARAS} by varying the size of the updates. The trend of frequent itemset mining module is similar to the trend of overall performance in Figure 16. The performance of the rule induction (updating) module, however, shows an opposite trend on this dataset. The times used of rule induction module are similar for \textit{iPARAS} and \textit{PARAS} when the size of the updates is small. E.g., the ratio is 86\% for 1\% update. The reason in this case is that even the size of the update was small, it brought significant changes to previous ruleset. The system is likely to search and make changes of the entire rule collection in the \textit{Adjacency Index} which made this process only a little faster than re-inducing from scratch. Yet the time saved becomes significant when the size of the updates increased. The optimized redundancy resolver in \textit{iPARAS}, as expected, significantly reduced the redundancy information abstracting time as compared to re-abstracting from scratch.

To evaluate \textit{iPARAS} on larger dataset, we used \textit{T20000K} for initial construction and \textit{T200K}, \textit{T400K}, \textit{T800K}, \textit{T1600K} for batch updates. For both \textit{iPARAS} and \textit{PARAS}, we set the primary support as 0.01\% and the primary confidence as 10\%. As shown in Figure 18, \textit{iPARAS} achieved about 62-fold, 42-fold, 21-fold and 12-fold speed-up of overall offline precomputing time with respect to the update size of 1\%, 2\%, 4\% and 8\%. The time differences decrease with increase of the size of the batch updates. Figure 19 shows the ratios of the running time of modules in \textit{iPARAS} to the running time of corresponding modules in \textit{PARAS} by varying the size of the updates. All three modules show similar trends as the
overall performances of iPARAS in Figure 18. Each of the modules significantly reduced
the precomputing time as compared to corresponding modules in PARAS.

5. Conclusion

We present our iPARAS framework for incremental parameter space construction to assure
online association rule mining. iPARAS corresponds to an end-to-end solution, composed of
three algorithms, for efficiently updating the precomputed ruleset for transaction databases.
In the context of the parameter space, we can achieve fast redundancy abstraction at offline
step by leveraging the proposed Adjacency Index. In a variety of tested cases, iPARAS
outperforms the overall performance of the state-of-the-art online association rule mining
systems, POQM (Aggarwal and Yu (2001)) and PARAS (Lin et al. (2013)). Our experi-
mental evaluation using benchmark datasets confirms that iPARAS achieves several orders
of magnitude improvement over the state-of-the-art approaches in online rule mining, as
well as dozen folds speed-up in offline construction in a variety of tested cases.

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